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Representation theoretic spectrum of locally symmetric spaces

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Abstract

The Laplace–Beltrami operator on a compact Riemannian manifold is self-adjoint with a discrete spectrum. Nevertheless, on a pseudo-Riemannian manifold, even a compact one, the situation is much more complicated: it is not clear whether the Laplace-Beltrami operator is self-adjoint, and its spectrum need not be discrete. We will focus on the case of double cosets $\Gamma\backslash G/H$ where G is a non-compact semisimple real Lie group, H is a closed subgroup such that the homogeneous space G/H is symmetric and Γ is a discrete subgroup of G such that $\Gamma\backslash G/H$ is a compact pseudo-Riemannian manifold. We will show that, even though the group G does not act on the double coset space $\Gamma\backslash G/H$, the representation theory of G nonetheless allows for an explicit description of the joint spectrum of the entire commutative algebra $D(G/H)$ of G-invariant differential operators, acting on $\Gamma\backslash G/H$ via unbounded operators. If time permits we will relate our results to Number theory and automorphic forms. A large part of the presentation will be devoted to explaining the ins and outs, as well as the motivations, behind the issue, notably through numerous examples. This talk is based on a joint work with Martin Olbrich.

Keywords: *Representation theory, Lie groups, Locally symmetric spaces, Laplace-Beltrami operator, Automorphic forms, Spectral analysis.*